**Cournot-Bertrand Competition: A Revisit of Strategic Trade Policy** **in the Third-Market Model**

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**Abstract**: Using Cournot-Bertrand duopoly models and incorporating the results of Brander and Spencer (1985) as well as Eaton and Grossman (1986), we arrive at a general, simple rule to determine the optimal policy of the home government for any combination of strategic variables: regardless of the strategic variable of the domestic firm, the optimal policy of the home country is an export subsidy (tax) as long as the foreign firm’s strategic variable is output (price). The optimal subsidy or tax of the home country is shown to move the equilibrium to the Stackelberg equilibrium where the domestic firm behaves as the leader while the foreign firm behaves as a follower under free trade. With appropriate interpretations and a suitable caveat, the above results still hold in the case with multiple foreign firms which may choose different strategic variables.

Keywords: strategic trade policy, strategic variable, Brander-Spencer model, rent-shifting

JEL Classification: F12, F13

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1. **Introduction**

The seminal papers of Spencer and Brander (1983) as well as Brander and Spencer (1985) have shown that R&D or export subsidies could under some conditions increase national welfare through rent-shifting if the domestic and foreign firms engage in Cournot output-setting competition in a third country market. Striking though, the robustness of the conclusion has been quickly rebutted by Eaton and Grossman (1986) simply by replacing Cournot output-setting competition with Bertrand price-setting competition. The three pioneering papers have ushered in a large body of literature of strategic trade policy, which extends the original models in various directions such as two active governments, domestic consumption of the exportable goods, multiple domestic and foreign firms, repeated games, and government budget constraint (Brander, 1995). Nonetheless, these works are flawed by a common denominator; namely, all of them assume that the two firms adopt the same strategic variable.[[4]](#footnote-4) More specifically, all the firms either set outputs or prices simultaneously. However, this kind of assumption is peculiar and unjustifiable both from the theoretical and from the real world points of view.

Consider the duopoly model as specified in Brander and Spencer (1985) or Eaton and Grossman (1986). Let the two firms be firm 1 and firm 2 and suppose that each can choose between two strategic variables, output and price. Then, there will be four possible combinations of strategies, (output, output), (output, price), (price, output), and (price, price), where the first (second) element in each vector is the strategic variable chosen by firm 1 (firm 2). It is evident that Cournot competition and Bertrand competition are nothing but two special cases out of the four possibilities. Theoretically, without further information concerning the characteristics of the firms, there is no *a priori* reason to prefer either Cournot or Bertrand to the rest two. In other words, it is perfectly likely that the two firms would choose different strategic variables in a duopoly market. As for what happens in the real world, it is useful to recognize the fact that an output setter is an unaggressive price taker while a price setter is an aggressive price maker.[[5]](#footnote-5) Depending on different characteristics of the firms in a given market, it is natural that some of them act more like aggressive price makers and the others more like unaggressive price takers. Therefore, the assumption that all firms act uniformly with respect to their choice of strategic variable makes not much sense in the real life. Perhaps for those reasons, some researchers have recently begun to consider the case where one of the duopolistic firms chooses output whereas the other chooses price as strategic variable, or Cournot-Bertrand competition (Naimzada and Tramontana, 2012; Ten Kate, 2014; Tremblay and Tremblay, 2011a; Tremblay et al. 2011b; Tremblay et al. 2011c). To the authors’ knowledge, however, there is still no study of strategic trade policy under the Cournot-Bertrand model. The main objective of this paper is to fill this gap in the literature, and thus provides some more general results not easily detected in the original model of Brander and Spencer or Eaton and Grossman.

The rest of the paper is organized as follows. Section 2 modifies the Brander-Spencer model so that the two firms can choose different strategic variables. After analyzing both the (Cournot, Bertrand) and (Bertrand, Cournot) cases, we combine the results with those of Brander and Spencer (1985) as well as Eaton and Grossman (1986) (hereinafter BS and EG) to arrive at a more general conclusion in Section 3. Section 3 also extends the duopoly models to the case with multiple foreign firms which may choose different strategic variables. Section 4 concludes the paper.

1. **The Models**

Our models follow all the assumptions of Brander and Spencer (1985) with only two modifications: (1) the two firms produce differentiated products, and (2) the two firms choose different strategic variables. As a result, each of our models is a two-stage game. In the first stage the home government decides the optimal export tax or subsidy to maximize social welfare, and in the second stage each firm chooses the level of its strategic variable to maximize its profit.

* 1. Domestic firm sets output and foreign firm sets price

Suppose a domestic firm (firm 1) and a foreign firm (firm 2) produce differentiated products, and, and export all their outputs to a third country where the prices of  and are and , respectively. Firm 1 (Firm 2) behaves as a Cournot-type output (Bertrand-type price) setter. Let the inverse demand functions be, , where  from now on. Making use of the inverse demand functions, the profit function of firm,, can be expressed in terms of the strategic variables  and  as:

, (1)

, (2)

where  is the fixed marginal cost of firm  and  () is the specific subsidy (tax) of the home government on the exports of firm 1.[[6]](#footnote-6)

Assuming that the second-order conditions hold, the first-order conditions for profit maximization are:

, (3)

. (4)

Equations (3) and (4) implicitly define the reaction functions,  and , of the two firms with slopes:

, (5)

.[[7]](#footnote-7) (6)

 is positively sloped because the marginal profit of firm 1, , increases as  rises, leading the optimal output level to increases. By the same token,  has a negative slope. The intersection of  and  determines the equilibrium (,).[[8]](#footnote-8) Figure 1 depicts such an equilibrium with the inverse demand functions  and , .

Substituting  and  into equations (3) and (4), we are able to see how changes in home government’s policy, , affect the strategic variables. Totally differentiating equations (3) and (4), we get:

 = . (7)

Solving (7) by Cramer’s rule gives us:

, (8)

, (9)

where  by the second-order conditions and signs of and . As expected, an increase (a decrease) in home government’s subsidy (tax) increases domestic firm’s optimal output level while depresses foreign firm’s optimal price.

Now let’s turn to the optimal policy of the home government, which is to maximize the country’s welfare. Since there is no domestic consumption, the welfare is the profit of firm 1 net of (plus) export subsidy (tax revenue). That is, . The first-order condition for welfare maximization is:

. (10)

Since  and,  by (9), we have:

. (11)

As a result, starting with free trade the home government can improve domestic welfare by taxing the exports of firm 1. This result, along with (8) and (10) gives us the home country’s optimal tax as follows:

. (12)

In other words, home government’s optimal policy is an export tax when the domestic firm sets output and the foreign firm sets price. This is depicted in Figure 2, where  and  denote equilibria of zero and the optimal export tax, respectively.[[9]](#footnote-9) Notice that firm 1’s iso-profit curve is tangent to firm 2’s reaction curve, , at , implying that  is a Stackelberg equilibrium under free trade when firm 1 behaves as a leader while firm 2 behaves as a follower.[[10]](#footnote-10)

* 1. Domestic firm sets price and foreign firm sets output

This case differs from what discussed in the last section only in that the two firms exchange their strategic variables. Consequently, we will not repeat all the details to save space. The profit functions in terms of the strategic variables  and  are:

, (13)

. (14)

The first-order conditions for profit maximizations are:

, (15)

. (16)

The reaction functions implicitly defined by (15) and (16) have slopes as follows:

, (17)

.[[11]](#footnote-11) (18)

Equations (15) and (16) determine the equilibrium (,), which is represented by the intersection of reaction curves  and  in Figure 3.

Making use of the first-order conditions, (15) and (16), and performing comparative statistics at the equilibrium, we obtain:

, (19)

, (20)

where, .[[12]](#footnote-12) As a result, an increase (a decrease) in home government’s subsidy (tax) not only depresses domestic firm’s optimal price but also decreases foreign firm’s optimal output level.

Now the government tries to choose the optimal value of  to maximize the welfare . The first-order condition is:

. (21)

That the two goods are substitutes implies  and thus. Combining this with (20), we have:

. (22)

In sharp contrast to the last case, the home government should subsidize the domestic firm to raise the domestic welfare when firm 1 sets price while firm 2 sets output. Finally, by (19), (20) and law of demand, equation (21) gives rise to the optimal subsidy:

. (23)

Figure 4 depicts the impact of home government’s optimal subsidy on the strategic variables as well as profits of the two firms. Without subsidy the reaction curves are  and, and  is the equilibrium. The curve  shifts downward whenever the home government subsidizes the exports of firm 1. The optimal subsidy is the one that moves  to  so that one of firm 1’s iso-profit curves is tangent to  at the intersection point , where both  and  are smaller than those at . However, in the present case the increase in the profit of firm 1 is at the cost of firm 2’s profit. Again, it can be shown that  is a Stackelberg equilibrium under free trade when firm 1 behaves as a leader while firm 2 behaves as a follower.

1. **Comparative Analysis and Extensions**
   1. Comparative analysis

We now put together what we obtained above and those of BS and EG to derive a more general conclusion about the optimal policy of the home country. The results are summarized in Table 1, which arranges the four combinations of strategies in two groups according to the strategic variable of firm 2. The first (bottom) two rows give the relevant results when firm 2 sets output (price). It is clear to see that the optimal policy of the home country is an export subsidy (tax) as long as firm 2’s strategic variable is output (price). Alternatively, the optimal policy of the home country depends on what firm 1 anticipates what firm 2 keeps fixed when firm 1 maximizes its profit. Therefore, we establish:

***Proposition 1****: Under the assumption that the two goods are substitutes, the optimal policy of the home country is an export subsidy (tax) as long as the foreign firm’s strategic variable is output (price)* *regardless of the strategic variable of the domestic firm.*

The result is perfectly in line with the conclusion of Ten Kate (2014): “What matters is not what you set, but what you assume the others keep fixed.” This is because firm 1 becomes a *de facto* monopolist in the residual market once firm 2 chose and fixed its output level. For a monopolist it makes no difference whether it chooses output or price to maximize its profit, implying that the optimal policy of the home government in the first stage stays the same. The same logic applies when firm 2 chose its price as it is tantamount to fixing some level of output. Consequently, it does not matter what firm 1’s strategic variable is as long as firm 2’s choice of strategic variable is made.

But, why should the home government subsidize (tax) domestic firm’s exports when the strategic variable of foreign firm is output (price)? Taking firm 2’s strategic variable is quantity as an example. If firm 1’s strategic variable is output, then its profit function becomes. Partially differentiate  with respect to  gives us:

. (24)

If firm 1’s strategic variable is price, then partially differentiate (13) with respect to  gives us:

. (25)

Equations (24) and (25) suggest that any policy leading to decreases of firm 2’s output would raise firm 1’s profit and thus the welfare of the home country. Consequently, the optimal policy of the home government must be an export subsidy as it depresses  under the assumption that the goods are substitutes. In a similar vein, it is readily to show that, when firm 2’s strategic variable is price, an export tax would raise  and increase firm 1’s profit as well as domestic welfare.

Proposition 1 provides us with a very simple, general rule to determine the optimal policy of the home government for all four combinations of strategic variables. However, it does not give us any information about the size of the optimum subsidy or tax. Fortunately, based on our results in Section 2 and those of BS and EG, we can prove:

***Corollary 1****: When the two goods are substitutes, the optimal policy of the home country moves the equilibrium to what would be the Stackelberg equilibrium where the domestic firm behaves as the leader while the foreign firm behaves as the follower under free trade.*

It is also noteworthy from Table 1 that firm 2’s output and profit decrease when the home government subsidizes exports of firm 1 (the first two rows). In other words, when firm 2’s strategic variable is output, the home government can take the advantage of the first-mover advantage vis-à-vis the firms to help firm 1 shift rent from firm 2 to maximize domestic welfare while hurt the welfare of foreign country. Conversely, firm 2’s price and profit increase when the home government taxes exports of firm 1 (the bottom two rows). Under the circumstances, imposing export tax by the home government facilitates collusion between the two firms, leading to higher profits (welfare) for both firms (countries) at the cost of the third country’s consumers.

* 1. Extensions

While simple and intuitively appealing, proposition 1 is based on Brander and Spencer’s original third-market duopoly model. Since the optimal policy depends exclusively on the foreign firm’s strategic variable, a natural question to ask is whether a similar conclusion can be obtained if there is more than one firm in the foreign country and not all of them choose the same strategic variable. Before proceeding to the general case, let’s consider two simplest cases where foreign country owns two firms, firm 2 and firm 3, with firm 2 (firm 3) choosing output (price) as its strategic variable while that of the firm 1 is output or price. It is evident that proposition 1 is not directly applicable here. Nevertheless, we might envisage that the result is very likely to depend on the relative strength of output effect of firm 2 and price effect of firm 3. This is what we would like to demonstrate below.

Case 1: Firm 1 sets output

As in the duopoly case and using similar notations, the profits of the three firms in terms of the strategic variable,  and  are, respectively:

, (26)

, (27)

. (28)

Let ,  and  denote the solutions of maximizing the three firms’ profits for given , then the home government shall choose  in the first stage to maximize the following welfare:

. (29)

The optimal export subsidy or tax turns out to be:

, (30)

where , ,, , and .

Comparing (30) with (12) and (23), we find that in each case the denominator captures the impact of export subsidy or tax on firm 1’s output, which is always positive.[[13]](#footnote-13) The first [second] term in the numerator looks exactly the same as that of (12) [(23)]. Loosely speaking, (30) can be regarded as the combination of (12) and (23). Since  stands for the impact of the government policy on the domestic firm’s marginal profit via the strategic variable of firm 3, a negative  implies that the optimal policy is an export tax. However, this is not the only effect in the present case; the government policy also affects the marginal profit of firm 1 via firm 2’s strategic variable. This effect is captured by the positive , suggesting an export subsidy as the optimal policy. As a result, the sign of (30) and thus the home government’s optimal policy is generally indeterminate. For simplicity, let us call  () the indirect price (output) effect. Then, whether the optimal policy is an export tax or an export subsidy depends on the relative strength of the two effects. It is an export subsidy (tax) if the indirect output (price) effect dominates the indirect price (output) effect. Last but not least, the  in (30) can be shown to move the equilibrium to what would be the Stackelberg equilibrium where firm 1 behaves as the leader and the two foreign firms behave as followers under free trade.

Case 2: Firm 1 sets price

Without repeating the details, let ,  and  be the solutions of maximizing the three firms’ profits for given , and the home government’s welfare maximization policy is:[[14]](#footnote-14)

, (31)

where , , , , and. Like (30), the numerator of (31) consists of the indirect output and price effects. However, a complication comes in the denominator that represents the effect of home government’s policy on firm 1’s output. While the first two terms are positive when the goods are substitutes, the third term turns out to be negative. This is a salient feature in any duopoly or oligopoly model like we have here with two firms using prices as their strategic variables.[[15]](#footnote-15) For the present 3-firm model, we follow the argument of Eaton and Grossman (1986) to assume that the first two terms dominate the third one so that the denominator of (31) is positive.[[16]](#footnote-16) Barring this caveat, the result is the same as that of Case 1: the optimal policy is an export subsidy (tax) if the indirect output (price) effect dominates the indirect price (output) effect, and it results in the Stackelberg equilibrium where firm 1 behaves as the leader and the two foreign firms behave as followers under free trade.

Finally, (30) and (31) can readily be extended to cases where there are  foreign firms, of which the first  (the rest ) firms choose outputs (prices) as their strategic variables.[[17]](#footnote-17) Depending on the strategic variable of firm 1, we have:

 , (30’)

. (31’)

Under similar circumstances discussed in (30) and (31), we thus establish:

***Proposition 2****: Under the assumption that all goods are substitutes and*  *foreign firms choose outputs (prices) as their strategic variables, the optimal policy of the home country is an export subsidy (tax) as long as the indirect output (price) effect dominates the indirect price (output) effect regardless of the strategic variable of the domestic firm.*

And

***Corollary 2****: When all goods are substitutes and*  *foreign firms choose outputs (prices) as their strategic variables, the optimal subsidy or tax of the home country moves the equilibrium to what would be the Stackelberg equilibrium where the domestic firm behaves as the leader while the n foreign firms behave as followers under free trade.*

It is now clear that the very general Proposition 2 and Corollary 2 subsume BS and EG as two special cases with  as well as  and , respectively.

1. **Conclusion**

A common but peculiar practice in a duopoly or oligopoly analysis is to assume that all firms choose the same strategic variable. In the case where two firms choose between setting output or setting price we end up with the celebrated Cournot (output, output) model and the Bertrand (price, price) model. While straightforward and intuitively appealing, this kind of practice is flawed both from the theoretical and from the real world points of view. Not only is there no *a priori* theoretical justification to exclude the possibility that firms may choose different strategic variables, but we cannot observe that firms indeed behave uniformly in the markets of real world. It is therefore interesting and important to bridge this knowledge gap and see what would happen when duopolistic or oligopolistic firms choose to use different strategic variables. This paper is intended to address the problem in the context of strategic trade policy using Brander and Spencer’s third-market model. Starting with the duopoly models and incorporating the results of BS and EG, we arrive at the very general, simple rule to determine the optimal policy of the home government for any combination of strategic variables: regardless of the strategic variable of the domestic firm, the optimal policy of the home country is an export subsidy (tax) as long as the foreign firm’s strategic variable is output (price). Moreover, the optimal subsidy or tax of the home country moves the equilibrium to what would be the Stackelberg equilibrium where the domestic firm behaves as the leader while the foreign firm behaves as the follower under free trade.

With appropriate interpretations and a suitable caveat, the above simple results are shown to hold when there is more than one firm in the foreign country, of which some choose outputs while others choose prices as their strategic variables. We have not dealt with cases where there are multiple firms in the home country as it is well-known that a sufficient large number of domestic firms might lead to an opposite result even in the relatively simple case where there is only one foreign firm and all of the firms set outputs simultaneously (Brander, 1995). Despite of this, we believe that the insight derived in this paper that it is what the foreign firms’ strategic variables that determine the home government’s optimal policy has significantly improved our understanding of the essence of strategic trade policy in the third-market model.

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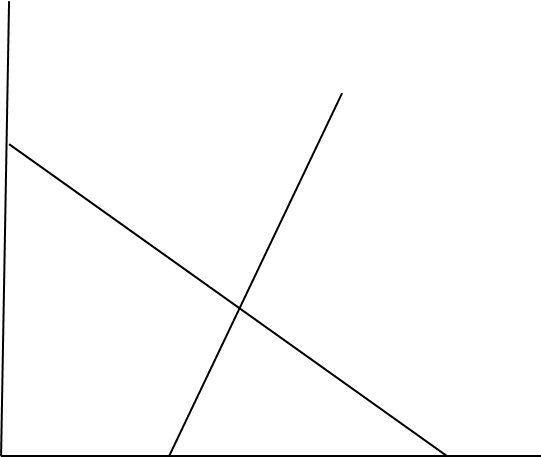




Figure 1: Reaction functions when firm 1 sets output and firm 2 sets price



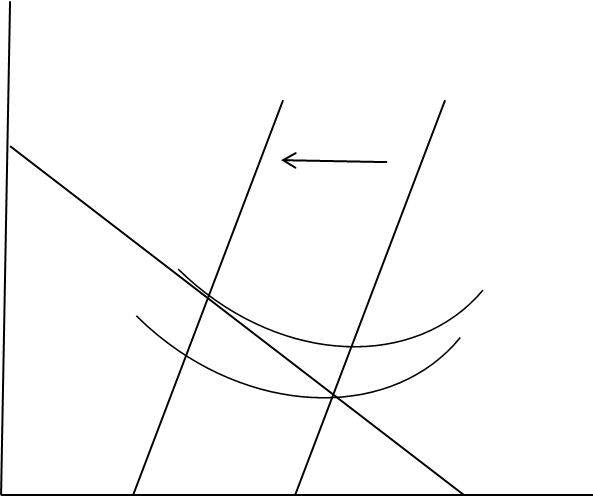


Figure 2: Optimal tax by home government when firm 1 sets output and firm 2 sets price



Figure 3: Reaction functions when firm 1 sets price and firm 2 sets output



Figure 4: Optimal subsidy by home government when firm 1 sets price and firm 2 sets output

Table 1: Comparative analysis of the four combinations of strategies

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Strategic variable | | Home optimal policy | Change in strategic variable | | Change in profit | |
| Firm 1 | Firm 2 | Firm 1 | Firm 2 | Firm 1 | Firm 2 |
| **Output** | **Output** | **Subsidy** |  |  |  |  |
| **Price** | **Output** | **Subsidy** |  |  |  |  |
| Output | Price | Tax |  |  |  |  |
| Price | Price | Tax |  |  |  |  |

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4. The argument is applicable to the case of multiple firms. However, for the sake of clarity, we will use the duopoly model in our illustration at this stage. [↑](#footnote-ref-4)
5. See Ten Kate (2014) for a cogent discussion on this point. It deserves mentioning that we discovered Ten Kate’s working paper, which came out on 21 August 2014, after we finished the first draft of the manuscript. Surprisingly, the arguments in Ten Kate (2014) are broadly in line with ours, though in quite a different context. [↑](#footnote-ref-5)
6. For simplicity, we assume no fixed costs of production. [↑](#footnote-ref-6)
7. It is easy to show that  and , where  and  when the two goods are substitutes. Note that = =0 if the demand functions are linear, though their signs are in general indeterminate. We will assume that sign () = sign () and sign () = sign () in our analysis. These assumptions along with the second-order conditions thus determine the slopes of  and . [↑](#footnote-ref-7)
8. For simplicity, we will always deal with the case with a unique, stable equilibrium. [↑](#footnote-ref-8)
9. It is easy to show that raising export tax will shift  leftward and that a higher iso-profit curve represents a higher profit of firm 1. [↑](#footnote-ref-9)
10. Proof of this and all other results below are available upon request from the authors. Moreover, it can be shown that the profits of both firms are higher at  than those at. Firm 2’s iso-profit curves are not shown in the figure to avoid jamming the figure. [↑](#footnote-ref-10)
11. Detailed assumptions and discussions concerning the slopes of  and  are available upon request from the authors. [↑](#footnote-ref-11)
12. The sign of  is determined by the argument similar to that of. [↑](#footnote-ref-12)
13. Of course, the numerical values are different. This is also true when we discuss about the two terms in the numerator. [↑](#footnote-ref-13)
14. The welfare function in this case is . [↑](#footnote-ref-14)
15. For instance, this also arises in Eaton and Grossman (1986) [↑](#footnote-ref-15)
16. This is always true in a linear demand model. However, the assumption becomes less likely to hold when more and more foreign firms have prices as their strategic variables as we will see below. [↑](#footnote-ref-16)
17. Noteshould not be so large that the strategic interactions among the  firms disappear. [↑](#footnote-ref-17)